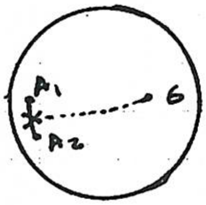

$$\left.\begin{matrix} X_{A_1} \\ X_{A_2} \\ X_G \end{matrix}\right\}$$ independent random points unit sphere S^{p-1}

$p=3$ { 3D sphere

$$t_a = \frac{\theta_2 R}{2 V_{A_1}}$$

V_{A_1} speed attacker 1

V_{A_2} speed attacker 2

$V_{A_1} = V_{A_2}$

$$t_g = \frac{\theta_1 R}{N V_{A_1}}$$

$V_c = V_{A_1} + V_{A_2}$

$V_c = V_{A_1} + V_{A_1} = 2 V_{A_1}$

$V_g = N V_{A_1}$

$t_a < t_g$ $\therefore$ attackers win

$$\frac{\theta_2 \cancel{R}}{2 \cancel{V_{A_1}}} < \frac{\theta_1 \cancel{R}}{N \cancel{V_{A_1}}}$$

$$\theta_2 < \frac{2\theta_1}{N}$$

$$h(\theta) = \frac{1}{\sqrt{\pi}} \frac{\Gamma\left(\frac{p}{2}\right)}{\Gamma\left(\frac{p-1}{2}\right)} \cdot (\sin\theta)^{p-2}$$

{ Distribution of angles in sphere n dimension.

$\theta \in [0, \pi]$

For R^3 & S^2, $p=3$

$$h(\theta) = \frac{1}{\sqrt{\pi}} \frac{\Gamma(3/2)}{\Gamma(1)} \cdot (\sin\theta)^{3-2}$$

$$h(\theta) = \frac{1}{\cancel{\sqrt{\pi}}} \frac{\cancel{\sqrt{\pi}}}{2(1)} \cdot \sin\theta$$

$$h(\theta) = \frac{1}{2} \sin\theta$$

$$P_{\substack{\text{attackers}\\ \text{wins}}} = \int_{\theta_1=0}^{\pi} \int_{\theta_2=0}^{2\theta_1/N} \frac{1}{2}\sin\theta_1 \, \frac{1}{2}\sin\theta_2 \, d\theta_1 \, d\theta_2$$

$$P_{\substack{\text{att.}\\ \text{wins.}}} = \frac{1}{4} \int_0^{\pi} \int_0^{2\theta_1/N} \sin\theta_1 \sin\theta_2 \, d\theta_1 \, d\theta_2$$

$$P_{\substack{\text{att}\\ \text{wins}}} = \frac{1}{4} \int_0^{\pi} \sin\theta_1 (-\cos\theta_2) \Big|_0^{2\theta_1/N} d\theta_1$$

$$P_{\substack{\text{att}\\ \text{wins}}} = \frac{1}{4} \int_0^{\pi} \sin\theta_1 \left[-\cos\left(\frac{2\theta_1}{N}\right) + \underbrace{\cos(0)}_{=1} \right] d\theta_1$$

$$= \frac{1}{4} \int_0^{\pi} \left[1 - \cos\left(\frac{2\theta_1}{N}\right) \right] \sin\theta_1 \, d\theta_1$$

$$= \frac{1}{4} \int_0^{\pi} \sin\theta_1 \, d\theta_1 - \frac{1}{4} \int_0^{\pi} \cos\left(\frac{2\theta_1}{N}\right) \sin\theta_1 \, d\theta_1$$

$$= \frac{1}{4} \left[-\cos\theta_1 \Big|_0^{\pi} \right] - \frac{1}{4} \int_0^{\pi} \cos\left(\frac{2\theta_1}{N}\right) \sin\theta_1 \, d\theta_1$$

$$= \frac{1}{4} \left[\underbrace{-(-1) + 1}_{=2} \right] - \frac{1}{4} \int_0^{\pi} \cos\left(\frac{2\theta_1}{N}\right) \sin\theta_1 \, d\theta_1$$

$$= \frac{1}{2} - \frac{1}{4} \int_0^{\pi} \cos\left(\frac{2\theta_1}{N}\right) \sin\theta_1 \, d\theta_1$$

$N = 20$

$$\Rightarrow \frac{1}{2} - \frac{1}{4} \int_0^{\pi} \cos(0.1\,\theta_1) \sin\theta_1 \, d\theta_1$$

$$\Rightarrow \frac{1}{2} - \frac{1}{4} \left[-0.55\bar{5} \cos(0.9\,\theta_1) - 0.455\bar{5} \cos(1.1\theta_1) \right]_0^{\pi}$$

$\Rightarrow$ $P_{\text{att. win}} = 0.00731$ $\quad P_{\text{guardian wins}} = 1 - 0.00731$
$= 0.99269 \cong 99.27\%$

Validated by Monte Carlo simulation.

$\cos x \cong 1 - \frac{x^2}{2!}$

$p = \frac{1}{2} - \frac{1}{4} \int_0^{\pi} \left(1 - \frac{1}{2!}\left(\frac{2\theta_1}{N}\right)^2\right) \sin\theta \, d\theta_1$

$\cancel{\frac{1}{2} - \frac{1}{4} \int_0^{\pi} \sin\theta \, d\theta_1}$ $- \frac{1}{4} \int_0^{\pi} \frac{1}{2!}\left(\frac{2\theta}{N}\right)^2 \sin\theta \, d\theta_1$

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$p = 0 - \frac{1}{\cancel{4}}\left(\frac{1}{\cancel{2}}\right)\frac{\cancel{2^2}}{N^2} \int_0^{\pi} \theta^2 \sin\theta \, d\theta_1$

$p = \frac{1}{2N^2} \int_0^{\pi} \theta^2 \sin \, d\theta_1$

$\int_0^{\pi} \theta^2 \sin\theta \, d\theta_1 = 2\theta \sin(\theta) - (\theta^2 - 2)\cos\theta \Big|_0^{\pi}$

$= 2\pi \cancel{\sin(\pi)}^{0} - 2(0)\cancel{\sin(0)}^{0} - (\pi^2 - 2)\cos\pi + (0 - 2)\cos 0$

$= -(\pi^2 - 2)(-1) + (0 - 2)1$

$= +\pi^2 - 2 - 2 = \pi^2 - 4$

$\therefore p_{\text{attack}} = \frac{\pi^2 - 4}{2N^2}$

~~gard~~ guardian wins $= 1 - \frac{\pi^2 - 4}{2N^2}$

guardian win $= 0.99266$

99.27 %

Same answer.